Beta splines, rational splines and computations

CS116B

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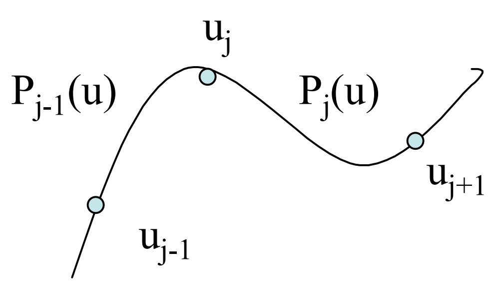
Outline

- Beta Splines
- Rational Splines
- Conversion Between Spline Representations
- Displaying Splines

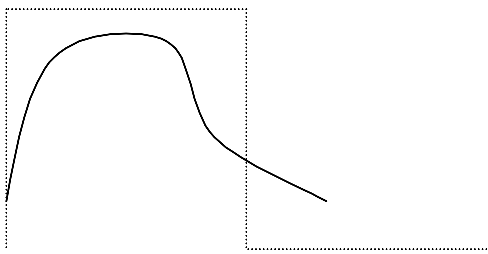
Beta Splines

- Beta-splines are a generalization of B-splines except we now have a geometric continuity conditions on the derivatives.

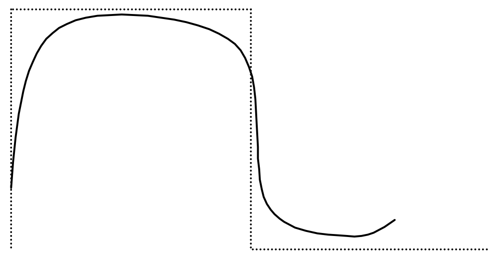


- Zeroth order continuity (G^0) is the condition that $P_{j-1}(u_j)= P_j(u_j)$. 1st order (G^1) continuity is that $b*P'_{j-1}(u_j)= P'_j(u_j)$ and 2nd order (G^2) continuity is that $b^2*P''_{j-1}(u_j) + c*P'_{j-1}(u_j)= P''_j(u_j)$.
- Here $b>0$ is called the *bias* and c is called the *tension*.

Example



b=1



b>>1

High b tends to flatten curve to the right

- Large values of c tend to make the curve hug its control graph more.

Cubic Periodic Beta-Spline Matrix Rperesentation

- Again, in the cubic case there is a matrix representation for Beta splines, $\mathbf{M_{Beta}}$=

$$\begin{bmatrix} -2b^3 & 2(c + b^3 + b^2+b) & -2(c+b^2+b+1) & 2 \\ 6b^3 & -3(c+2b^3 + 2b^2) & 3(c+2b^2) & 0 \\ -6b & 6(b^3+b) & 6b & 0 \\ 2b^3 & c + 4(b^2 + b) & 2 & 0 \end{bmatrix}$$

- The B-spline matrix is the special case where b=1 and c=0.

Rational Splines

- A *rational funtion* is a ratio of two polynomials.
- A *rational spline* is thus somehow (not exactly) the ratio of two splines.
- For example, to define a rational B-spline we use the equation:

$\mathbf{P}(u) = \sum^{n}_{k=0}\omega_k\mathbf{p}_k B_{k,d}(u)/\sum^{n}_{k=0}\omega_k B_{k,d}(u)$

where ω_k is a weighting factor affecting how close the curve is to the control point.

Advantages of Rational Splines

- Can give an exact representation of the different conic sections with rational splines
- They are invariant with respect to the perspective viewing transformations.
  – So to change the perspective only need to apply the perspective transformation to the control points.

Representations of rational splines

- Most graphics packages:
  - use non-uniform control points. So get NURBS (nonuniform rational B-splines).
  - Use homogeneous coordinates representations.
  - Otherwise,rational splines constructed in a similar way to usual B-splines.
- As an example, if d=3, {0,0,0,1,1,1} and $\omega_0 = \omega_2 = 1$ and $\omega_1 = r/(1-r)$

  $\mathbf{P}(u) = (\sum^{n}_{k=0} \mathbf{p}_0 B_{0,3}(u) + [r/(1-r)]\, \mathbf{p_1} B_{1,3}(u) + \mathbf{p_2} B_{2,3}(u)) / (B_{0,3}(u) + [r/(1-r)]\, B_{1,3}(u) + B_{2,3}(u))$
- When r>1/2 get hyperbola,when r=1/2 get a parabola, r<1/2 get an ellipse, when r=0 get a straight line.

Conversion Between Spline Representations

- Sometimes it is useful to be able to convert from one kind of spline to a different kind.
- For example, to convert from a B-spline representation to an equivalent Bezier spline one.
- Suppose have equations: $\mathbf{P(u) = U M_{spline1} M_{geom1}}$ and $\mathbf{P(u) = U M_{spline2} M_{geom2}}$.
- They represent the same curve if they are equal. Solving for $\mathbf{M_{geom2}}$ gives $\mathbf{M^{-1}_{spline2} M_{spline1} M_{geom1}}$.
- Here $\mathbf{M_{spline2,spline1} = M^{-1}_{spline2} M_{spline1}}$ does not depend on the control points. The book gives for Bezier curve to B-spline conversions.

Displaying Splines

- To display a spline curve involves evaluating the parametric polynomial splines for different values of u.
- So it is useful to know some efficient way to evaluate polynomials.
- A first trick is to use Horner's rule: To evaluate polynomials like a*u^3+b*u^2+c*u+d compute: ((a*u+b)*u +c)*u +d.

More on displaying splines

- x(u), y(u), z(u) must be calculated for successive values of u. Let's call these u_k.
- We assume $u_{k+1} = u_k + \delta$.
- Then $x_{k+1} = p(u_k + \delta)$ and $\Delta x_k = x_{k+1} - x_k = p(u_k + \delta) - p(u_k)$ for some polynomial p.
- Δx_k is called a forward difference. It will in general be a deg(p)-1 polynomial in $u_{k.}$
- Now we could in turn compute the forward difference of Δx_k. This would be a deg(p)-2 polynomial and allow us to compute successive values of Δx_k. Can keep going till get degree 0 polynomial, then have completely determined the problem.