On the edge-graceful \((n, kn)\)-multigraphs conjecture

Yong-Song Ho, Sin-Min Lee, Eric Seah

Department of Mathematics
National University of Singapore
REPUBLIC OF SINGAPORE

Department of Mathematics and Computer Science
San Jose State University
San Jose, California 95192
U.S.A.

Department of Actuarial and Management Sciences
University of Manitoba
Winnipeg, Manitoba R3T 2N2
CANADA

Abstract. Lee conjectures that for any \(k > 1\), a \((n, nk)\)-multigraph decomposable into \(k\) Hamiltonian cycles is edge-graceful if \(n\) is odd. We investigate the edge-gracefulness of a special class of regular multigraphs and show that the conjecture is true for this class of multigraphs.

1. Introduction.

The study of edge-graceful simple graphs was initiated in 1985 by Lo [13]. A simple graph \(G = (V, E)\) is said to be edge-graceful if there exists a bijection \(f : E \to \{1, 2, \ldots, |E|\}\) such that the induced mapping \(f^* : V \to \{0, 1, \ldots, |V| - 1\}\), defined by \(f^*(v) = \sum f(u, v) : (u, v) \in E(G) \pmod{|V|}\), is a bijection.

The concept of edge-graceful graphs can be viewed as the dual concept of graceful graphs. A graph \(G = (V, E)\) is graceful if there exists an injection \(g : V(G) \to \{0, 1, \ldots, |E|\}\) such that the induced mapping \(g^* : E \to \{1, 2, \ldots, |E|\}\), defined by \(g^*(a, b) = |g(a) - g(b)|\) for all \((a, b)\) in \(E\), is a bijection. Graceful graphs were considered by Rosa [14] in the early 60's and popularized by Golomb [3]. For further details on graceful graphs and their applications, we refer the readers to [1], [3], and [4].

In this paper, we extend the concept of edge-gracefulness for simple graphs to multigraphs. All the multigraphs considered herein have no loops. A multigraph \(G\) is said to be regular if for any \(u, v\) in \(V(G)\), we have \(d_G(u) = d_G(v)\).

The second author has conjectured that for any \(k > 1\), a \((n, nk)\)-multigraph decomposable into \(k\) Hamiltonian cycles is edge-graceful if \(n\) is odd in 1988 South-eastern International Conference on Combinatorics, Graph Theory, and Computing. The conjecture is true for several families of \((n, nk)\)-graphs, such as \(k\)th power cycles [9], complete graphs [6], and regular complete \(k\)-partite graphs [10].