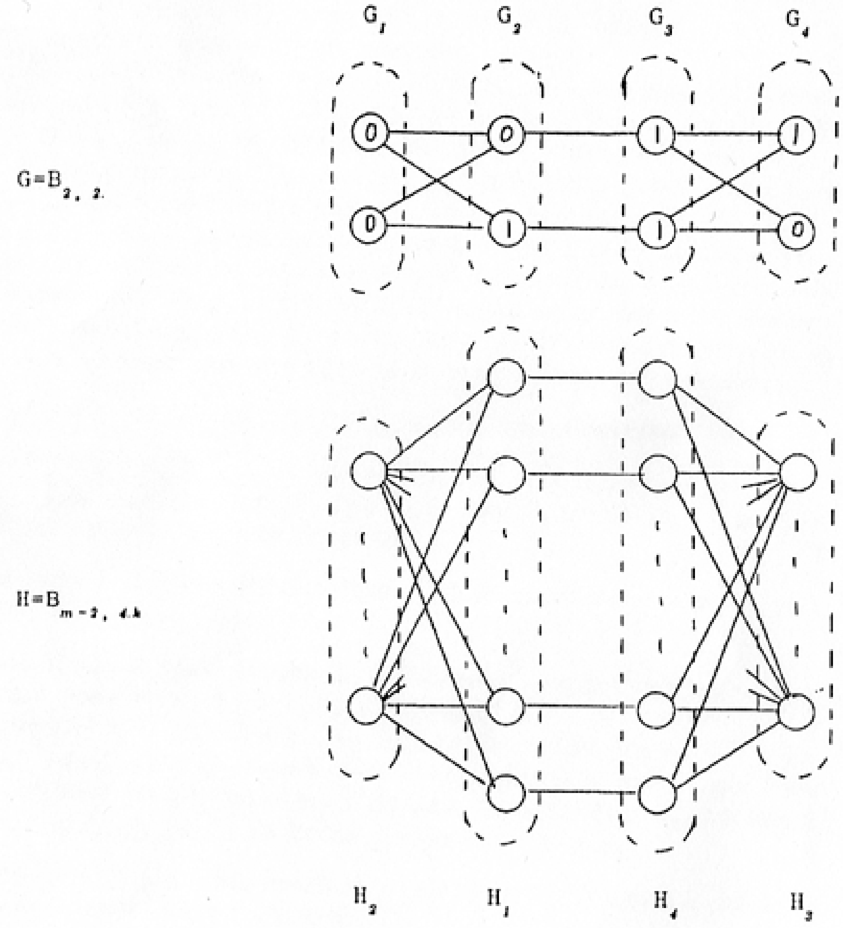

Figure 2. The construction of a cordial graph.

ON EXACT MINIMAL (1,5)-COVERS OF THIRTEEN POINTS

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Abstract. In "On the exact minimal (1,4)-cover of twelve points" (Ars Combinatoria 27, 3–18, 1989), Sane proved that if E is an exact minimal (1,5)-cover of thirteen points, then E has 282, 287, 292, or 297 blocks. Here we rule out the first possibility.

0. Introduction.

All the preliminary knowledge necessary to understand this paper can be found in [3, 4, 5, 6]. A (exact) $(1,t,v)$-cover D is a collection of proper subsets (blocks) of a v-set such that every t-tuple of elements (points) is contained in precisely one block of D. D is called a minimal cover if no other $(1,t,v)$-cover has fewer blocks. We write $|D|$ for the cardinality, that is, the number of blocks of D and $g(1,t,v)$ for $|D|$ when D is a minimal $(1,t,v)$-cover (or $(1,t)$-cover of v points). The packing number $D_k(v)$ is the largest number of k-subsets of a v-set such that no two k-subsets (blocks) intersect in $k-1$ points.

In [4], Sane proved that if E is a minimal $(1,5)$-cover of thirteen points then $|E| = 282, 287, 292$, or 297. The main purpose of the present paper is to rule out $|E| = 282$. We do this by proving (in Theorem 6) that if for some fixed point x, the derived cover E_x is a $(1,4)$-cover of twelve points, then $|E| = 297$ and E is precisely as described in [4]. We then prove (in Corollary 7) that if E is a minimal $(1,5,13)$-cover, then $|E| = 287, 292$, or 297. Our result is shown using the following possibly well known observation: An $(11,5,2)$ symmetric design has a unique embedding in the Steiner system $S(4,5,11)$, that is, the 55 blocks of the Steiner system $S(4,5,11)$ that are not in the $(11,5,2)$ symmetric design are uniquely determined as point-subsets by the blocks of the $(11,5,2)$ symmetric design. We believe that our elementary proof of this observation (Proposition 1) is perhaps of some independent interest.

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