one part with at least two vertices. Suppose two vertices in this part have opposite labels. Let $H = H_1$ consist of these two vertices, G be the graph induced by the remaining vertices and G_1 consist of all vertices not in the same part as H. Then (G, H) is the original graph, which is assumed to be cordial. By Theorem 1, G is also cordial. However, H is also a complete k-partite graph with at least 4 odd parts, contradicting the minimality assumption.

It follows that all vertices in each part of the original graph must have the same label. Hence, $e(1) = v(0)v(1)$ and $e(0) \le \frac{1}{2}v(0)(v(0)-1) + \frac{1}{2}v(1)(v(1)-1)$. Since $(v(0)-v(1))^2 \le 1$, we have $1 \ge e(1) - e(0) \ge \frac{1}{2}(v(0) + v(1) - (v(0)-v(1))^2) \ge \frac{1}{2}(v(0) + v(1) - 1)$. It follows that $3 \ge v(0) = v(1)$. However, with at most 3 vertices, the graph cannot have at least 4 odd parts. This completes the proof of the Theorem. ∎

Cahit's consideration of cordial graphs is motivated by the study of graceful graphs. This is a subject with a vast literature, and we will not discuss it here. A very enjoyable and informative account is given in [2].

Cahit regards cordial graphs as a weaker version of graceful graphs, although there are cordial graphs which are not graceful. While it is not known whether all trees are graceful, (5) gives the affirmative answer that they are all cordial. Theorem 4 completely solves the problem of which generalized bundles are cordial. In [3], where these graphs are called mirror-sums, only a partial answer to the problem of which of them are graceful is obtained.

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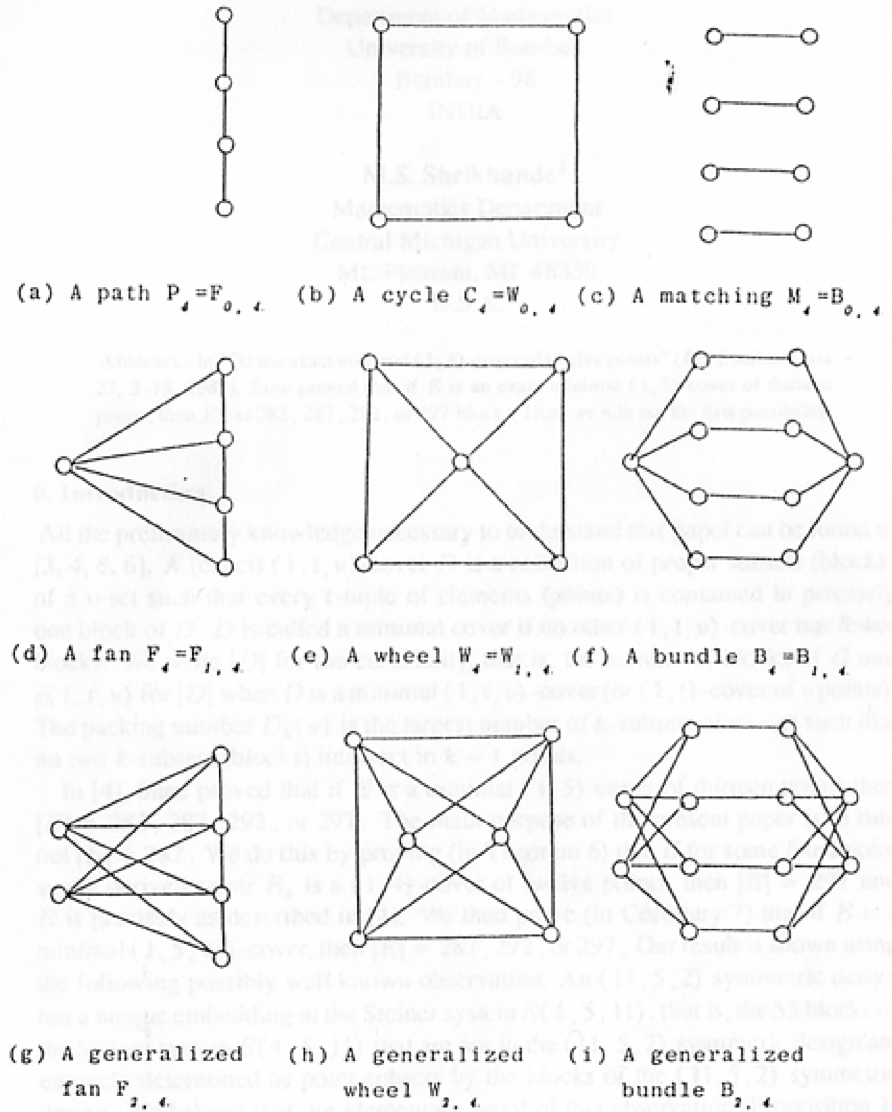

Figure 1. Some classes of graphs.