A CONSTRUCTION OF CORDIAL GRAPHS FROM SMALLER CORDIAL GRAPHS

S.M. Lee
Department of Mathematics and Computing Science
San Jose State University
San Jose, CA 95192

A. Liu[1]
Department of Mathematics
University of Alberta
Edmonton, ALTA, T6G 2G1

A binary labelling of a connected graph assigns 0 or 1 to each vertex of the graph, 0 to an edge joining two vertices having the same label, and 1 to an edge joining two vertices having opposite labels. For such a labelling, let $v(0)$, $v(1)$, $e(0)$ and $e(1)$ denote, respectively, the numbers of vertices labelled 0, vertices labelled 1, edges labelled 0 and edges labelled 1.

Cahit [1] defines a graph to be cordial if it has a binary labelling such that $|v(0) - v(1)| \leq 1$ and $|e(0) - e(1)| \leq 1$. We list below some of the results proved in that paper.

(1) In any binary labelling of a Eulerian graph, $e(1)$ is even.

(2) A Eulerian graph is not cordial if it has a number of edges congruent to 2 (mod 4).

(3) The complete graph K_n is cordial if and only if $n \leq 3$.

(4) All complete bipartite graphs are cordial.

(5) All trees (see Figure 1(a) for a special case) are cordial.

(6) The cycle C_n (see Figure 1(b)) is cordial if and only if $n \not\equiv 2 \pmod 4$.

(7) The matching M_n (see Figure 1(c)) is cordial if and only if $n \not\equiv 2 \pmod 4$.

(8) All fans (see Figure 1(d)) are cordial.

(9) The wheel W_n (see Figure 1(e)) is cordial if and only if $n \not\equiv 3 \pmod 4$.

In this paper, we prove additional results via the following construction.

Theorem 1. *Let H be a graph with an even number of edges and a cordial labelling such that the vertices of H can be divided into ℓ parts $H_1, H_2, \ldots, H_\ell$, each consisting of an equal number of vertices labelled 0 and vertices labelled 1. Let G be any graph and $G_1, G_2, \ldots, G_\ell$, be any ℓ subsets of the vertices of G. Let (G, H) be the graph which is the disjoint union of G and H augmented by edges joining every vertex in G_i to every vertex in H_i, $1 \leq i \leq \ell$. Then G is cordial if and only if (G, H) is.*

Proof: The given cordial labelling of H and any cordial labelling of G induce a binary labelling of (G, H). Since $v(0) = v(1)$ for H and $|v(0) - v(1)| \leq 1$

[1] Supported by NSERC grant A5137.