Technique 1: For a full-class $A_{u}$, we label edge $(u, u+i)$ by label $u$, where $u = 0, 1, ..., nk-1$. Note that the collection of these edge labels is the set $B = \{0, 1, ..., nk-1\}$. Now consider any vertex $u$: it has two incident edges, $(u, u+i)$ and $(u-i, u)$. The sum of the two edge labels is $u + (u-i) \pmod{nk}$, or $2u - i \pmod{nk}$.

Technique 2: For a full-class $A_{i}$, we label edge $(u, u+i)$ by label $(nk-u-1)$, where $u = 0, 1, ..., nk-1$. Note again that the collection of the edge labels form the set $B$. Consider any vertex $u$: it has two incident edges $(u, u+i)$ and $(u-i, u)$. The sum of the two edge labels is $(nk-u-1) + (nk-u+i-1)$, or $2nk-2u+i-2 \pmod{nk}$, or $-2u+i-2 \pmod{nk}$.

Observe that if we label two full-classes $A_{i}$ and $A_{j}$ by Techniques 1 and 2 respectively, the sum of (four) edge labels for all vertices are identical and equal to $-i+j-2 \pmod{nk}$. In other words, these two full-classes give the same contribution to all vertex labels.

4. Case (1) -- Odd $n$ and Odd $k$

In this section, we consider the labelings for the easy case of odd $n$ and odd $k$. Recall that in this case there are $n(k-1)/2$ full-classes (and no half-classes). There are two sub-cases to consider:

(1.1) The number of classes, $n(k-1)/2$, is even:

Here we label any $n(k-1)/4+1$ full-classes by Technique 1, and the remaining $n(k-1)/4-1$ full-classes by Technique 2. Note that we can form $n(k-1)/4-1$ pairs of full-classes, such that the two full-classes in each pair are labeled by Techniques 1 and 2 respectively. These $n(k-1)/4-1$ pairs of full-classes have the same contribution (say f) to the vertex labels (see Section 3). Without loss of generality, let the two remaining full-classes labeled by Technique 1 be $A_{i}$ and $A_{j}$. Consider any vertex $u$, the vertex label is given by $[2u - i] + [2u - j] + f \pmod{nk}$, or $4u - i - j + f \pmod{nk}$. It is easy to see that the vertex labels for two distinct vertices are not identical.

(1.2) The number of classes, $n(k-1)/2$, is odd:

We label any $[(n(k-1)/2)-1]/2+1$ full-classes by Technique 1, and the remaining $[(n(k-1)/2)-1]/2$ full-classes by Technique 2. Similar to case (1.1) above, we can form $[(n(k-1)/2)-1]$ pairs of full-classes, which give the same contribution (say f) to the vertex labels. That leaves us with a class (say $A_{i}$) labeled by Technique 1. Now consider any vertex $u$, the vertex label is given by $2u - i + f \pmod{nk}$, which is again distinct.