A LABELING ALGORITHM FOR MAGIC GRAPH

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Abstract

A graph $G(p, q) = (V, E)$ is magic if we can find an edge labeling assignment $L: E \rightarrow \{1, 2, \ldots \}$ such that the sum of all edge labels incident to each vertex has the same value, and this value is called the magic index $w$.

In this paper we present an efficient labeling algorithm for a magic graph. Given a graph, the algorithm uses a modified depth-first search strategy to label the graph. If the graph is magic, there exists at least one value of $w \leq q$. Heuristics based on properties of magic graph are employed to terminate the search for a magic labeling if the graph is not magic and to reduce the search space if the graph is magic.

1. Introduction

Let $G$ be a graph having $p$ vertices and $q$ edges, i.e., if $G = (V, E)$, then $|V(G)| = p$ and $|E(G)| = q$. The induced map $L' : V \rightarrow N$, also called the vertex sum, is defined by $L'(v) = \sum fL(u, v) :$ for all $(u, v) \in E$. A labeling, $L: E \rightarrow \{1, 2, \ldots \}$, is called magic with index $w$ if the vertex sum of each vertex is $w$.

The concept of a magic graph was introduced first by J. Sedlacek [7, 8], where he used label values as distinct non-negative real numbers. The question of the precise structure of such labeling has been investigated from several viewpoints. Stewart [9] looked at the properties of the real vector space coordinatized by the edges. Doob [3] considered the label values in an abelian group or a ring. Jeurissen [5] called a magic labeling pseudo magic if the labels were pairwisely distinct. Berge [1] called a graph regularisable if a regular multigraph could be obtained from $G$ by adding edges parallel to the edges of $G$. In fact, a graph is magic if and only if it is regularisable. Berge showed that the necessary and sufficient condition for a connected non-bipartite graph $G$ to be regularisable is that $|N(S)| > |S|$ for every nonempty independent set of vertices $S$, where $N(S)$ is the set of neighbors of vertices in $S$. Jaeger and Payan [4] characterized regularisable graphs without $K_{1,2}$ as an induced subgraph. Recently, Lee, Saba and Sun [6] introduced the concept of magic strength and discussed the magic strength of the $kth$