

**Case 3.**  $\{f(A), f(B), f(C)\} = \{\text{even, even, even}\}$ . It is obvious that  $|E_e| = |V_{me}| + |V_{ne}| + |V_{te}|$ . Assume  $|V_{me}| = i$  and  $|V_{ne}| = j$ , then  $|V_{te}| = (\lfloor k/2 \rfloor + 2) - i - j - 3$ . So  $|E_e| = i + j + (\lfloor k/2 \rfloor + 2) - i - j - 3$ , we simplify it to get  $|E_e| = \lfloor k/2 \rfloor - 1$ . This is a contradiction because  $|E_e| = \lfloor k/2 \rfloor$ .

**Case 4.**  $\{f(A), f(B), f(C)\} = \{\text{odd, odd, odd}\}$ . It is obvious that  $|E_e| = |V_{mo}| + |V_{no}| + |V_{to}|$ . Assume  $|V_{mo}| = i$  and  $|V_{no}| = j$ , then  $|V_{to}| = (\lceil k/2 \rceil + 1) - i - j - 3$ . So  $|E_e| = i + j + (\lceil k/2 \rceil + 1) - i - j - 3$ , we simplify it to get  $|E_e| = \lceil k/2 \rceil - 2$ . This contradicts to the fact that  $|E_e| = \lfloor k/2 \rfloor$ .

### 3. Remarks

We conclude with some questions which remain to be answered, and some conjectures for Skolem-graceful graphs. What are known thus far are that

- (1) A 2-star  $St(n_1, n_2)$  is Skolem-graceful if and only if  $n_1 n_2$  is even.
- (2) A 3-star  $St(n_1, n_2, n_3)$  is Skolem-graceful if and only if not all  $n_1, n_2$ , and  $n_3$  are odd.

One may lead to the following conjecture:

**Conjecture:** A 4-star  $St(n_1, n_2, n_3, n_4)$  is Skolem-graceful if and only if not all  $n_1, n_2, n_3$ , and  $n_4$  are odd.

However, the conjecture is false for  $n_1 = n_2 = n_3 = n_4 = 1$ . For we have a Skolem sequence of rank 4 which provide a Skolem-graceful labeling of  $St(1, 1, 1, 1)$ .

Thus, the problem of characterization of Skolem-gracefulness of 4-stars is still unsolved. It is conjectured in [2] that all 4-stars are Skolem-graceful. Several theorems showing Skolem-gracefulness of some 4-stars are given in [2]. However, a complete solution is lacking.

We propose the following

**Problem.** For any  $n \geq 4$ , give a characterization of Skolem-graceful  $n$ -stars.

### References

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4. Th. Skolem, On certain distributions of integers in pairs with given differences, *Math. Scandinavica* 5 (1957), 57-68.