Theorem 3. $C_n$ is felicitous if:

(i) $n$ is odd, or
(ii) $n = 4k$, $k = 1, 2, \ldots$

Proof. (i) $C_{2k+1}$ is harmonious [4].
(ii) Let the vertices of $C_{4k}$ be $v_0, u_0, v_1, u_1, \ldots, v_{2k-1}, u_{2k-1}$. We label $u_i$ and $v_i$ by $f(u_i)$ and $f(v_i)$ respectively, where

$$f(u_i) = \begin{cases} 
4k & \text{for } i = 0, \\
i & \text{for } 1 \leq i < k, \\
i + 1 & \text{for } i \geq k,
\end{cases}$$

$$f(v_i) = \begin{cases} 
0 & \text{for } i = 0, \\
2k + i & \text{for } i = 1, 2, \ldots, 2k - 1.
\end{cases}$$

Example 2. A felicitous labelling of $C_5$ and $C_{12}$ are shown in Fig. 3(a) and (b) respectively.

Let $S_m$ be a star with $m$ edges. Let $C_n \odot S_m$ (resp. $C_n \otimes S_m$) be a graph obtained by identifying any vertex of $C_n$ with the center (resp. any vertex not the center) of $S_m$.

Theorem 4. $C_{2k+1} \odot S_m$ and $C_{2k+1} \otimes S_m$ are felicitous for all $k$ and $m$.

Proof. For $C_{2k+1} \odot S_m$, we label the identified vertex with $k$, the cycle with $k, 0, k + 1, 1, \ldots, k - 1, 2k$ and the remaining vertices with $2k + 1, 2k + 2, \ldots, 2k + m$. 

For example, $C_9 \odot S_4$ is labelled felicitously as shown in Fig. 4(a).

For $C_{2k+1} \otimes S_m$, we label the identified vertex with $m + k + 2$, the cycle with $m + k + 2, 2, m + k + 3, 3, \ldots, k + 1, 1$, the center of $S_m$ with 0, and the remaining vertices with $k + 3, k + 4, \ldots, m + k + 1$. 

![Fig. 3](attachment:image.png)