


## Nussinov Algorithm

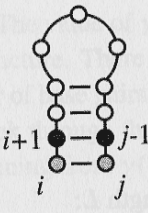
- Given an RNA sequence of length  $L$ .
  - Find the structure with the most base pairs
- There are four ways of getting the best structure between position  $i$  and  $j$  from the best structures of the smaller subsequences:
  - 1) Add unpaired position  $i$  onto best structure for subsequence  $i+1, j$
  - 2) Add unpaired position  $j$  onto best structure for subsequence  $i, j-1$
  - 3) Add  $i, j$  pair onto best structure found for subsequence  $i+1, j-1$
  - 4) Combine two optimal structures  $i, k$  and  $k+1, j$

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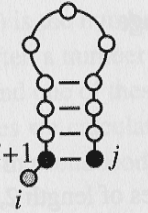
## Nussinov Algorithm: Four Cases

**3**



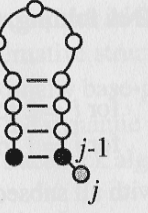
$i, j$  pair

**1**



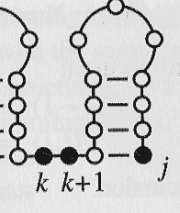
$i$  unpaired

**2**



$j$  unpaired

**4**



bifurcation

The numbers above each case corresponds to the enumeration on the previous slide.

Biological Sequence Analysis by Durbin et al., 2001, page 269

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## Nussinov Algorithm: The Table

- Compares a sequence against itself in a dynamic programming matrix.
- Since structure folds upon itself, it is only necessary to calculate half the matrix.
- The value of  $M[i,j]$  is the number of base pairs in the maximally base-paired structure.
- Four rules for scoring the structure at a particular point.

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## Nussinov Algorithm: Initialization

- **Initialization:** fill the main diagonal and the diagonal just below it with zeros
- Formally, the scoring matrix,  $M$ , is initialized:
  - $M[i,i] = 0$  for  $i = 1$  to  $L$  (main diagonal)
  - $M[i,i-1] = 0$  for  $i = 2$  to  $L$  (diagonal below main diagonal)

Biological Sequence Analysis by Durbin et al., 2001, page 270

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## Nussinov Algorithm: Recursion

Matrix Fill: Starting with all subsequences of length 2, to length L


**do**

$M[i,j] = \max$  of the following :

- $M[i+1,j]$  (*base i is hanging off by itself*)
- $M[i,j-1]$  (*base j is hanging off by itself*)
- $M[i+1,j-1] + S(x_i, x_j)$  (*bases i and j are paired; if  $x_i = \text{complement of } x_j$ , then  $S(x_i, x_j) = 1$ ; otherwise it is 0*)
- $M[i,j] = \text{MAX}_{i < k < j} (M[i,k] + M[k+1,j])$  (*merging two substructures*)

Biological Sequence Analysis by Durbin et al., 2001, page 270  
Adapted from leah.haifa.ac.il/~zfrenkel/Sasha/CB/

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## Nussinov: Traceback Stage

Push (1,L) onto stack (Start with this coordinate: i=1, j=L)

Recursion: Repeat until stack is empty

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-pop (i,j);
if i >= j continue;

else if M(i+1,j) = M(i,j) then push (i+1,j);

else if M(i,j-1) = M(i,j) then push (i,j-1);

else if M(i+1,j-1) = M(i,j) + d[i,j] then
  record base i,j;
  push (i+1,j-1);

else for k = i+1 to j-1: if M(i,k) + M(k+1,j) = M(i,j) then
  push (k+1,j);
  push (i,k);
  break;

```

Biological Sequence Analysis by Durbin et al., 2001, page 271

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