San José State University
Department of Computer Science
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Spring 2014
CS 255
Due: March 13, 2014

Assignment THREE

Problem 1

Consider a 1 by n chessboard. Suppose that we color each square of the chessboard in either blue or red. Let $T(n)$ represent the number of possible colored chessboards in which no two squares that are colored red are adjacent.

a) Find a recurrence relation for $T(n)$.

b) Find the closed form of $T(n)$.

Problem 2

A) Which of these codes cannot be Huffman codes for any probability assignment? Why?

a) {0, 10,11}.

b) {00, 01, 10, 110}.

c) {01, 10}.

B) Given symbols a, b, c, d, e, f (in this order) having frequencies 0.3, 0.2, 0.2, 0.1, 0.1, 0.1, respectively, construct a binary Huffman code and calculate L, the average code word length of the obtained code. Make sure to show the resulting tree in your solution.

Problem 3

Let $T(h, k)$ be the maximum number of leaves of a tree of height h, where each node has outdegree (number of children) k or less.

a) Find a recurrence relation for $T(h, k)$.

b) Solve the recurrence relation.

Problem 4

Solve the recurrence relation

$$nT(n) + nT(n-1) - T(n-1) = 2^n$$
$$T(0) = 273$$

Problem 5

Explain in your own words Theorem 30.8 on page 914.

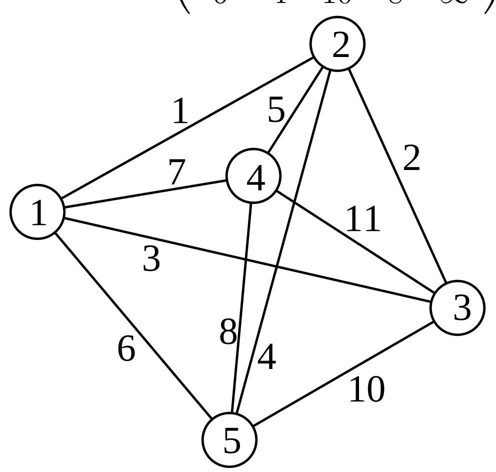
Problem 6

Extra credit. $a = (10, 11, 12, 13, 14, 15, 16, 17)$ is a polynomial given in coefficient representation. You are going to trace through algorithm $RecurFFT(a, n)$, given on page 22 of the lecture notes, to show how it computes the discrete Fourier transform of a. Use your creativity and imagination to show the details of every step. No partial credit given for the extra credit problem.

Problem 7

Use the dynamic programming algorithm we studied to solve the following instance of TSP. Recall that the algorithm computes values $P(S, k)$, for $S \subseteq \{2, 3, \ldots, n\}$ and $k \in S$, that represent the cost of a shortest path that starts at 1, visits all vertices in $S \setminus \{k\}$, and ends at k. For $|S| > 1$, the value of $P(S, k)$ is calculated as $\min\{P(S \setminus \{k\}, m) + w(m, k) \mid m \in S \setminus \{k\}\}$.

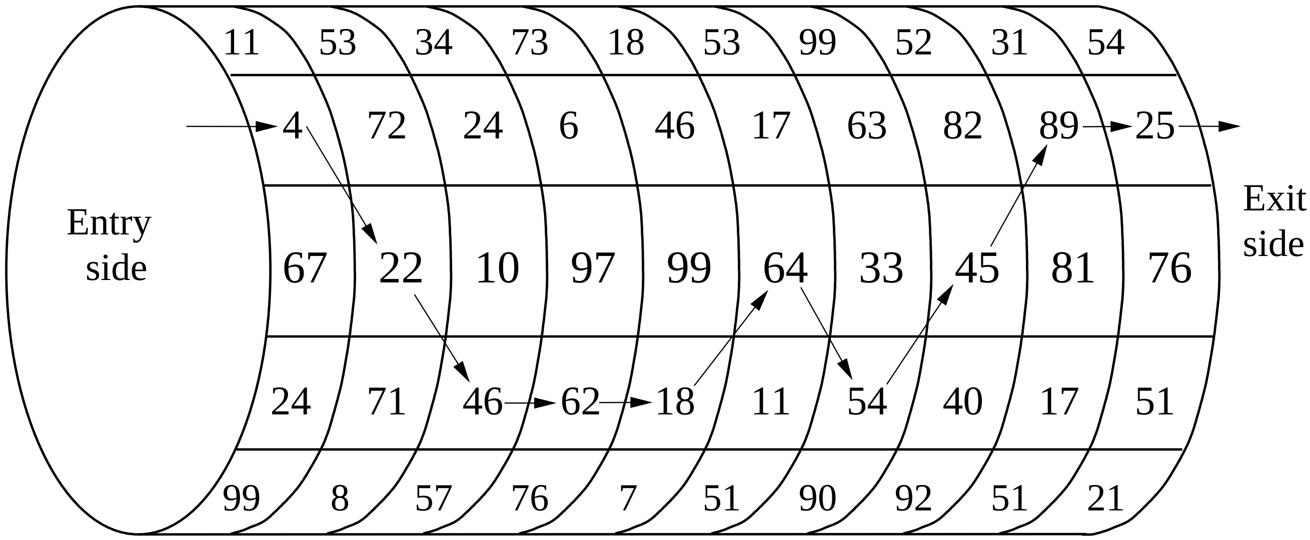
$$\text{cost matrix: } \begin{pmatrix} \infty & 1 & 3 & 7 & 6 \\ 1 & \infty & 2 & 5 & 4 \\ 3 & 2 & \infty & 11 & 10 \\ 7 & 5 & 11 & \infty & 8 \\ 6 & 4 & 10 & 8 & \infty \end{pmatrix}$$



Also, define a $Pred(S, k)$, for each $P(S, k)$, to be the value of "m" which minimizes the expression in calculating $P(S, k)$. Use these values $Pred(S, k)$ to find the optimal Hamiltonian circuit.

Problem 8

Consider an $n \times n$ array of positive integers (a_{ij}), $0 \leq i, j < n$, rolled into a cylinder, as illustrated below.



A path is to be threaded from the entry side of the cylinder to the exit side, subject to the restriction that from a given square it is possible to go only to one of the three positions in the next column adjacent to the current position. The path may begin at any position on the entry side and may end at any position on the exit side. The cost of such a path is the sum of the integers in the squares through which it passes. Thus the cost of the sample path shown above is 429.

(a) How many distinct paths are there?

(b) Show how we can use the dynamic programming technique to find the optimal path (cheapest path).

(c) Find the growth rate of the running time function of your solution to (b).