
Assignment TWO

Problem 1

Exercise 2.3-3 page 39.

Problem 2

Exercise 4.3-1 page 87.

Problem 3

Exercise 4.5-1 page 96.

Solve all four recurrence relations but instead of using the master method described in the textbook, use the one we went over in class.

Problem 4

Problem 4-1 page 107.

Note that some of the recurrence relations stem from divide and conquer algorithms while others do not. Once again, when needed, use the master method we went over in class.

Problem 5

True or False? In each case, give the proof supporting your claim. Show all your work.

a) $\frac{1}{\omega_n^k} = \overline{\omega_n^k}$, where the bar represents complex conjugation, in other words:

$$\overline{a + ib} = a - ib.$$

b) $\omega_n = \omega_{kn}^k$. In other words, is every n^{th} root of unity a $(kn)^{\text{th}}$ root of unity?

c) $\omega_6 = \frac{1}{2}(1 - i\sqrt{3})$.

d) For $n = 4$, the Vandermonde matrix defined in the Appendix of the lecture notes (on page 11) is given by

$$V_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$$

e) Go to page 11 of the Appendix and start by writing the Vandermonde Matrix for $n = 16$. Can it be simplified to obtain V_{16} whose entries have only $1, -1, i, -i$ such as V_4 of part d?

Problem 6

Exercise 30.2-2 page 914.

Extra Credit: Compute the DFT using a different method.

Problem 7

Show that the n complex 5^{th} roots of unity form a group under multiplication. Show all your work.

Problem 8

Trace through the commutative diagram of Figure 30.1, page 904 (also in the lecture notes) to show how the Fourier Transform is used. Use the following two functions: $f(x) = 2 - x$ and $g(x) = 4 + 3x$ and show all your work.

Problem 9

Solve the following recurrence relations:

- $T(n) - 7T(n - 1) + 10T(n - 2) = 3^n, \quad T(0) = 0, \quad T(1) = 1$
- $T(n) + 6T(n - 1) + 9T(n - 2) = 3, \quad T(0) = 0, \quad T(1) = 1$
- $T(n) + T(n - 1) + T(n - 2) = 0, \quad T(0) = 0, \quad T(1) = 2$