

Assignment Five

Please hand in the solutions to the following problems on Thursday, May 1, 2008.



Problem 1 Nuts and Bolts

We have a collection of n bolts of different sizes, and n corresponding nuts. We can test whether a nut and bolt are an exact match or whether the nut is too large or too small for the bolt. We can not directly compare two nuts or directly compare two bolts since the differences in sizes between 2 nuts or 2 bolts are too small to see by the naked eye.

- (a) Give an algorithm that correctly pairs up the n nuts and bolts in $O(n^2)$.
- (b) We are interested in finding the smallest bolt and its corresponding nut.
 - i) What is the minimum number of comparisons needed (best case scenario)
 - ii) What is the maximum number of comparisons needed (worst case scenario)

Problem 2

There are only two buttons inside an elevator in a building with 50 floors. The elevator goes 11 floors up if the first button is pressed and 6 floors down if the second button is pressed.

Is it possible to get from floor 32 to floor 33?

- a) If it is possible:
 - i) What is the minimum number of buttons one has to press?
 - ii) What is the shortest time one needs to get from floor 32 to floor 33 (where time is proportional to the number of floors that are passed on the way).
- b) If it is not possible, explain why not.

[An Introduction to Bioinformatics Algorithms by Jones and Pevzner, 2004].

Problem 3

We are given 3 strings of characters: X , Y , and Z , where $|X| = n$, $|Y| = m$, and $|Z| = n + m$. Z is said to be a shuffle of X and Y iff Z can be formed by interleaving the characters from X and Y in a way that maintains the left-to-right ordering of the characters from each string.

- a) Show that $Z_1 = sjojsub$ is a shuffle of $X = sjsu$ and $Y = job$.
- b) Show that $Z_2 = sjjusob$ is not a shuffle of $X = sjsu$ and $Y = job$.
- c) Give an efficient algorithm that determines whether Z is a shuffle of X and Y .

Problem 4

Exercise 34.4-2 page 1002.

Note that formula 34.3 is on page 999.

Hint: Show all the intermediate steps.

In other words, show ϕ' , ϕ'' and ϕ''' .

Problem 5

HITTING-STRING is the language that contains $\langle S \rangle$, where S is a finite set $\{s_1, s_2, \dots, s_k\}$ of equal-length strings over $\Sigma = \{0, 1, *\}$, each having the same length n . $\langle S \rangle$ is in the language if there is a "hitting string" for S , i.e., a string $y \in \{0, 1\}^n$ with the property that for each $s_t \in S$, there is some $1 \leq i \leq n$ for which the i th character of s_t and the i th character of y are identical.

Example: 010 is a hitting string for $\{0*1, 100, 110, **0\}$; and $\{*01, 111, **0, 011\}$ admits no hitting string.

Show that HITTING-STRING is NP-complete
Hint: Use SAT (satisfiable formulae in CNF).